Effects of QCD Vacuum and Stability of H Dihyperon *

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Within the composite quark model taking into account the interaction of quarks in the bag with vacuum fields of QCD the masses of H ($I = 0, S = -2, J = 0, 1^C$) and H^* ($I = 0, S = -2, J = 1, 1^C$) dihyperons are estimated: $M_H = 2.16$ GeV, $M_{H^*} = 2.34$ GeV. It is shown that the leading effect giving a stable with respect to strong decays H dihyperon is the instanton interaction forming diquarks: $q^2(J = 0, C = \underline{\bar{3}})$, $q^2(J = 0, C = \underline{6})$. With the approach developed the contribution of QCD hyperfine interaction is suppressed, and instanton induced three-particle forces in multiquark hadrons ($q^3(J = 1/2, C = \underline{8})$ - channel) are rather small.

1. Within the quark bag model [1, 2, 3] the consideration of multiquark states [4, 5, 6, 7] was one of the most interesting applications. In these calculations the MIT version of bag model was used. As proposed in a static approximation [1], the energy of a multiquark system is determined [2, 8] by

$$E\left(R\right) = \sum_{flav} \frac{n_i \omega_i}{R} + \frac{4\pi}{3} B R^3 - \frac{Z_0}{R} + \Delta E_g, \tag{1}$$

$$M^2\left(R\right) = E^2 - \left\langle P^2 \right\rangle, \qquad \left\langle P^2 \right\rangle \simeq \sum_{flav} n_i \left(\frac{\varkappa_i}{R}\right)^2, \tag{2}$$

where R is the bag radius, n_i is the number of quarks of an i-th type with energy ω_i/R ($\omega_{u,d} = 2.043$ in the ground s-state; $m_u = m_d = 0$, $m_s \cong 280$ MeV), B is an external pressure ($B^{1/4} \cong 145$ MeV), $-Z_0/R$ is a contribution of zero-mode fluctuations ($Z_0 = 1.84$), ΔE_g represents a color-magnetic interaction ($\alpha_s = 2.2$). The stability condition $\frac{dM^2}{dR} = 0$ fixes the bag radius.

In the MIT version of the model the hadron spectrum is specified by the QCD interaction

$$\Delta E_g = -\frac{\alpha_s}{4R} \sum_{i>j}^{N} \mu\left(m_i R, m_j R\right) \left(\overrightarrow{\sigma} \lambda^a\right)_i \left(\overrightarrow{\sigma} \lambda^a\right)_j, \tag{3}$$

where N is the total number of quarks, $\overrightarrow{\sigma}_i$ (λ_i^a) are the spin (color) operators of an i-th quark; $\mu_{ij} \equiv \mu\left(m_i R, m_j R\right)$ determine the strength of a color-spin interaction. In a massless ($m_i = 0$) case the average of (3) over a hadron state q^n is expressed through the Casimir operators of spin SU_2^J: $\frac{4}{3} J(J+1)$; color SU_3^C: $C_3 = 0$; spin-color SU_6^{CJ} - defined by quantum numbers of the considered state

$$\Delta E_g = \mu_{00} \left[8N - \frac{1}{2} C_6 + \frac{4}{3} J(J+1)\right] \frac{\alpha_s}{4R}, \tag{4}$$

where J is the total moment. Based on this formula it is possible to formulate the rules [4] analogous to the Hund rules from atomic physics.

For the case of dibaryons rules are the following [5]: The lightest states are those in which quarks are in the most symmetrical (antisymmetrical) with respect to color-spin (flavor) representations.

Such general considerations and calculations based on the MIT model lead to the conclusion [5] that a flavor-singlet six-quark dihyperon H with strangeness -2 and $J^P = 0^+$ may be stable with respect to a strong decay.

Recently, the problem of stability of the H dihyperon has again been discussed after unusual signals from the Cygnus X-3 have been registered [9]. As a possible explanation of this effect it was proposed [10] that Cygnus X-3 is a star containing the strange matter and it emits H dibaryons with a lifetime $\tau_H \gtrsim 10$ years. The value of H mass is an essential point in the determination of its lifetime and confirmation of this hypothesis [11]. While today experimental

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situation on the emission from Cygnus X-3 stays indefinite [12], the search for multiquark states in cosmic rays and on accelerators intensively continues [13]. That is why correct calculations of multiquark masses, their lifetimes and decay modes are so important. In this respect the H dihyperon is most intriguing object of the investigations.

The mass of H was estimated within different variants of the bag model [5, 6, 14], the lattice approach, the QCD sum rules method and Skyrme model [15]. The aim of the present work is to calculate the mass of H in the quark bag model taking into account the structure of QCD vacuum [16]. In [16] such a model was proved to be consistent with the method of QCD sum rules and capable of describing the spectroscopy of the ground states of hadrons.

2. Let us formulate the basic assumptions of the model [16]. It is known [2] that in the MIT model, it is assumed conserning the vacuum structure that in the presence of valence quarks nonperturbative vacuum fully goes out of the bag. However, this hypothesis is not compatible with the picture produced by the QCD sum rules [17], and it is not self-consistent. Really, the bag constant B characterizing the degree of destroying of vacuum $B_{\mathrm{MIT}} \sim (130 - 150 \quad \mathrm{MeV})^4$ [18] is much smaller than the "depth" of nonperturbative vacuum known from the QCD sum rules: $\varepsilon_0 = -\frac{9}{32} \left\langle 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 \right\rangle \simeq -(240 \quad \mathrm{MeV})^4$. This means that the vacuum fields practically do not change inside the bag-hadron. Therefore, the neglect of the effects of QCD vacuum in the bag becomes physically groundless.

In works [16] basic principles were formulated allowing one to consider the QCD vacuum in the bag.

As a starting point, we take the QCD theory and available information on the behavior of its solution. First, we think that in a zero approximation the structure of the solution is such that the low- and high-frequency components of the solution are independent of each other.

Second, we suppose that the (valence) components of the fields with characteristic frequencies $\omega \sim \omega_q$ are described by solutions of the static bag-model equations $\left(q(x) = q^{\mathrm{bag}}(x); \quad A_{\mathrm{T}}(x) = A_{\mathrm{T}}^{\mathrm{bag}}(x) \right)$.

Third, low-frequency (condensate) field components ($\omega << \omega_q$) are assumed to be solutions of the QCD equations characterized by a set of numbers: different vacuum condensate quantities ($\langle \overline{Q}Q \rangle$, $\langle G^2 \rangle$,...). Under these assumptions the Hamiltonian of the interaction of valence components $(q(x), A_{\mathrm{T}}(x))$ with condensate ones $(Q(x), A_{\mathrm{vac}}(x))$ is restored uniquely through the field transformation:

$$\Psi(x) = q(x) + Q(x); \qquad A(x) = A_{\mathrm{T}}(x) + A_{\mathrm{vac}}(x). \tag{5}$$

In addition, there are QCD vacuum fluctuations with $\omega_{\mathrm{vac}} >> \omega_q$ which may be approximated by the 't Hooft interaction [19] induced by instantons.

3. In the model [16] the energy of a hadron is defined as

$$M^2 = E^2 - \left\langle P^2 \right\rangle, \qquad \frac{dM^2}{dR} = 0, \tag{6}$$

where $\left\langle P^2 \right\rangle \simeq \sum_{\mathrm{flav}} n_i \left(\varkappa_i / R \right)^2$ is due to the c.m. motion of quarks [8] and

$$E(R) = E_{\mathrm{kin}} + \Delta E_{\mathrm{g}} + \Delta E_{\mathrm{vac}} + \Delta E_{\mathrm{inst}} \tag{7}$$

is the bag energy. In (7) the kinetic energy of quarks E_{kin} and the one-gluon interaction energy ΔE_{g} are calculated as usually in the bag perturbation theory [1, 18, 20]:

$$E_{\mathrm{kin}} = \sum_{\mathrm{flav}} n_i \frac{\omega_i}{R}, \tag{8}$$

$$\Delta E_{\mathrm{g}} = \frac{0.117\alpha_s}{R} \left[M_{00} + (1 - 0.13 m_s R) M_{0s} + (1 - 0.25 m_s R) M_{ss} \right], \tag{9}$$

where M_{ij} denotes matrix elements of the operator (2) with respect to spin-color spin states of hadrons.

As was shown in [16] a leading contribution to the hadron energy caused by the valence- and condensate-fields interaction is generated by the Hamiltonian

$$H_{\mathrm{vac}} = \frac{\omega_q}{2} \left(\overline{Q} \gamma^0 q + \overline{q} \gamma^0 Q \right). \tag{10}$$

Then by using stationary perturbation theory

$$\Delta E_{\mathrm{vac}} = \frac{\langle \Phi | H_I | \Psi \rangle}{\langle \Phi | \Psi \rangle}, \qquad | \Psi \rangle = U(-\infty, 0) | \Phi \rangle, \tag{11}$$

$$U\left(-\infty,0\right) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{0} dt_1 ... \int_{-\infty}^{t_{n-1}} dt_n T\left[H_{\text{vac}}\left(t_1\right)...H_{\text{vac}}\left(t_n\right)\right],$$

where $|\Phi\rangle$ is a nonperturbed hadron wave function of the bag model, we have [16]

$$\Delta E_{\text{vac}} = -n_0 \frac{\pi}{24} \frac{\langle 0\,|\overline{u}u|\,0\rangle}{\varkappa_0 - 1} R^2 - n_s \frac{\pi}{12} \frac{\langle 0\,|\overline{s}s|\,0\rangle}{\varkappa_s^2} R^2 \frac{(y+a)^2 y}{2y(y-1)+a} + \tag{12}$$
$$+ \frac{\pi^2}{1152} \frac{\langle 0\,|\overline{u}u|\,0\rangle^2}{\varkappa_0\left(\varkappa_0 - 1\right)^2} R^5 \left\{ \widetilde{M}_{00} + \frac{(\varkappa_0 + y)(y+a)^2(\varkappa_0 - 1)}{\varkappa_s^2\left[2y(y-1)+a\right]} \frac{\langle 0\,|\overline{s}s|\,0\rangle}{\langle 0\,|\overline{u}u|\,0\rangle} \widetilde{M}_{s0} + \right.$$
$$\left. + \frac{4y(y+a)^4 \varkappa_0\left(\varkappa_0 - 1\right)^2}{\varkappa_s^4\left[2y(y-1)+a\right]^2} \left(\frac{\langle 0\,|\overline{s}s|\,0\rangle}{\langle 0\,|\overline{u}u|\,0\rangle}\right)^2 \widetilde{M}_{ss} \right\} + ...,$$

where $\langle 0\,|\overline{Q}_i Q_i|\,0\rangle$ are quark condensates, $y = \omega_s R$, $a = m_s R$.

Expressions (12) are absolutely different from BR^3 arising *ed hoc* in the MIT version. In contrast, in the model considered stability of the bag is achieved in a self-consistent manner due to the interaction of quarks with a physical vacuum. Moreover, the potential (12) is drastically dependent on the number of quarks with a given flavor, their masses and quantum numbers of considered hadron states (the latter is taken into account by the coefficients $\widetilde{M}$ [16]).

So, the long-wave vacuum fluctuations define the effective quark mass (12). At the same time the interaction of quarks with a short-wave part of vacuum fluctuations allows us, to a great extent, to explain the mass splitting between the terms of $SU_f(3)$ hadron multiplets [16]. Within the model of QCD vacuum as an instanton liquid [21] we get the two-particle contribution to the energy [16]

$$\Delta E_{\text{inst}}^{(2)} = -n_0 \sum_{\substack{a>b \\ \text{flav}}} \eta_{ab} I_{ab} \left\{ 1 + \frac{3}{32} \lambda_a \lambda_b \left(1 + 3\overrightarrow{\sigma}_a \overrightarrow{\sigma}_b\right) \right\} \tag{13}$$

and the contribution of three-particle forces for multiquark states

$$\Delta E_{\text{inst}}^{(3)} = +n_0 \eta_{uds} I_{uds} \left\{ 1 + \frac{3}{32} \lambda_a \lambda_b \left(1 + 3\overrightarrow{\sigma}_a \overrightarrow{\sigma}_b\right) + \right. \tag{14}$$
$$- \frac{9}{320} d^{\alpha\beta\gamma} \lambda_u^\alpha \lambda_d^\beta \lambda_s^\gamma \left[1 - 3\left(\overrightarrow{\sigma}_a \overrightarrow{\sigma}_b + \text{permutations}\right)\right] +$$
$$\left. - \frac{9}{64} f^{\alpha\beta\gamma} \varepsilon_{ijk} \left(\sigma^i \lambda^\alpha\right)_u \left(\sigma^j \lambda^\beta\right)_d \left(\sigma^k \lambda^\gamma\right)_s \right\}.$$

Here $n_0 = \langle 0\,|g^2 G^{a\mu\nu} G_{\mu\nu}^a|\,0\rangle / (64\pi^2)$ is a density of instantons in the model [21] $\left(n_0 = \left(\pi\rho_c \langle \overline{Q}Q\rangle\right)^2 /3\right)$,

$$\eta_{i_1...i_n} = \left(\frac{4}{3}\pi^2 \rho_c^3\right)^n / \left(m_{i_1}^* \rho_c ... m_{i_n}^* \rho_c\right),$$

$$m_i^* = m_i + m^*, \qquad m^* = -\frac{2\pi^2}{3} \langle 0\,|\overline{u}u|\,0\rangle \rho_c^2,$$

ρ_c is the characteristic size of an instanton in the QCD vacuum,

$$I_{a_i...a_n} = \int_{\text{bag}} d\overrightarrow{r} \prod_{i=1}^{n} \overline{q}_R^{a_i} q_L^{a_i}.$$

It should be emphasized that the interaction through instantons (13), (14) takes place in a system $|\rangle$ of quarks in a zero mode [16, 19, 22, 23, 24]

$$\sum_{i=1}^{n} \left(\overrightarrow{\sigma}_S^i + \overrightarrow{\tau}_C^i\right) |\rangle = 0. \tag{15}$$

Diquarks: $q^2 \left(\overline{\underline{3}}^F, J = 0, \overline{3}^C \right)$, $q^2 \left(\overline{\underline{3}}^F, J = 1, 6^C \right)$, triquarks: $q^3 \left(\underline{8}^F, J = \frac{1}{2}, 8^C \right)$, $q^3 \left(J = \frac{3}{2}, 10^C \right)$, *etc.* are such systems.

We also note that the instanton interaction (13), (14) is consistent only in the first order of perturbation theory analogously to that as it was done in the case of an external pion field [25].

4. The model parameters.

As is seen from (1), the MIT model uses four parameters (B, α_s, Z_0, m_s). But, the parameter Z_0 is not well grounded, the value of m_s is too large, and B poorly agrees with the parameters extracted from the QCD sum rules and current algebra. The value $\alpha_s = 2.2$ does not agree with a perturbative expansion in this parameter, which was confirmed in one-loop calculations [26]. In multiquark systems, perturbative calculations with large α_s get still more uncertain [6].

Within the description [16] of energies of the ground states of hadrons it has been proved that it suffices to choose the following values of the parameters

$$\alpha_s = 0.7, \qquad m_s = 220 \quad \text{MeV}, \tag{16}$$

and $\rho_c = 2$ GeV^{-1}, $\langle 0 | \overline{Q}_i Q_i | 0 \rangle = - (250 \quad \text{MeV})^3 \quad (i = u, d, s)$ adopted from the model of vacuum [21] and QCD sum rules [17], respectively.

5. To calculate the matrix elements of two- and three-particle operators included into ΔE_g, ΔE_{vac}, $\Delta E_{\text{inst}}^{(2)}$, $\Delta E_{\text{inst}}^{(3)}$ it is necessary to know the cluster expansion (dissociation) of the six-quark wave function of H: $q^6 \to q^3 \times q^3$, $q^6 \to q^4 \times q^2$. The expansion method and wave functions are given in Appendix.

By using the wave functions (A.3) we have for the matrix elements ΔE_g, $\Delta E_{\text{inst}}^{(2)}$:

$$\Delta E_g^H = \left(-5\mu_{00} - 22\mu_{0s} + 3\mu_{ss} \right) / R, \tag{17}$$

$$\Delta E_g^{H^*} = \frac{1}{3} \left(\frac{11}{6}\mu_{00} - 41\mu_{0s} + \frac{67}{6}\mu_{ss} \right) / R,$$

$$\Delta E_{\text{inst}}^{(2)H} = -\frac{27}{4} n_0 \left(I_{ud}\eta_{ud} + 2I_{us}\eta_{us} \right), \tag{18}$$

$$\Delta E_{\text{inst}}^{(2)H^*} = -\frac{43}{8} n_0 \left(I_{ud}\eta_{ud} + 2I_{us}\eta_{us} \right).$$

In accordance with the selection rule (15) the three-particle interaction is nonzero only for the component $q^3 \left(I = 0, S = -1, J = 1/2, \underline{70}^{CJ} \right)$. So, with the wave function (A.3) we have:

$$\Delta E_{\text{inst}}^{(3)H} = +\frac{135}{8} n_0 I_{uds}\eta_{uds}. \tag{19}$$

It should be added that the approximation of coefficients μ is given in [20]; coefficients I_{ij} in [16] $(I_s\eta_s \simeq 0.65 I_0\eta_0)$, $I_{uds} = 0.0244/R^6$.

6. By using the above-mentioned relations we obtain the estimation of dihyperon masses

$$M_H = 2.16 \quad \text{GeV}, \qquad R_H = 5.2 \quad \text{GeV}^{-1}, \tag{20}$$

$$M_{H^*} = 2.34 \quad \text{GeV}, \qquad R_{H^*} = 5.3 \quad \text{GeV}^{-1}.$$

So, our results show that the mass of H is less than $2M_\Lambda$ but above the threshold of $N\Lambda$. Dihyperon H^* is absolutely unstable: $M_{H^*} > 2M_\Lambda$.

In accordance with the estimation of the lifetime of H in $\Delta T = 1$ weak decays established in work [27], the state with the mass $M_H = 2.16$ GeV is long-lived: $\tau_H \sim 10^{-8}$ sec.

Note that in the approach developed a basic cause of the stability of H dihyperon is the interaction of valence quarks with short-wave fluctuations and physically is due to the same mechanism by which the mass splitting arises in hadron multiplets ($\pi - \rho$, $N - \Delta$, and so on splittings).

We also proved the spectroscopic Hund rule for quark systems [4, 5, 6]. At the same time its origin is absolutely different. The nonperturbative instanton interaction between a pair of quarks produces strong attraction in a symmetric in color-spin representation and is totally absent in antisymmetric states. The instanton interaction takes into account the strong interaction at intermediate distances ρ_c and gives use to the formation of quasibound states, diquarks [23, 28]. So, the Hund rule is physically due to the existence of diquarks.

Moreover, in multiquark system there are multiparticle $(n > 2)$ instanton-induced forces (in colorless baryons such interactions are absent because of the selection rule (15)). At the same time we show that the contribution of three-particle forces to the energy of H state is rather small, $\Delta E_{\text{inst}}^{(3)} \sim +70$ MeV.

Note also that very recently in the experiment carried by the B.A. Shahbasian group the data that confirm the existence of H dihyperon have been obtained [29].

Acknowledgments

Appendix. The wave function of dihyperons.

To construct the wave functions of H and H^* dihyperons, we make use of the dissociation method developed in [30]. The idea of this method was borrowed from work [6]. That method allows us to construct the wave functions of multiquark systems $q^m \overline{q}^n$ with respect to arbitrary dissociation $(q^{m_1}\overline{q}^{n_1}) \times (q^{m_2}\overline{q}^{n_2})$ $(m_1 + m_2 = m, n_1 + n_2 = n)$. To classify the basis states of multiquarks (q^m), the group U_{18} is chosen as the group in which the direct production of isospin SU_2^I, strangeness U_1^S, spin SU_2^J and color SU_3^C groups is embedded. (Singling out the flavor group SU_3^F is not effective because of a strong mixing of the SU_3^F quantum numbers arising in the multiquark sector.) Thus, the quark wave function relative to their quantum numbers transforms by a fundamental representation $\{1\}$ of the group U_{18}. In this case the scheme of the group reduction of the n-quark-system representation $\{1^n\}$ is the following

$$
\begin{aligned}
U_{18}^{\times n} \to U_{18} \to SU_{12}^{IJC} \times U_6^{SJC} \to & \qquad \text{(A.1)} \\
\to \left(SU_2^I \times SU_6^{JC}\right) \times \left(U_1^S \times SU_6^{JC}\right) \to & \\
\to SU_2^I \times \left(SU_2^J \times SU_3^C\right) \times U_1^S \times \left(SU_2^J \times SU_3^C\right) \to & \\
\to SU_2^I \times U_1^S \times SU_2^J \times SU_3^C. &
\end{aligned}
$$

Using Racah's factorization lemma (1949) and the factorization property of transformation coefficients for direct product groups, the transformation of a 6-quark wave function to the dissociation basis may be written in the form [30]

$$
\left| \left(1^3, 1^3\right) 1^6 \left(1^{n_0} \left(I, \lambda_0^{JC}, J_0\mu_0^C\right), 1^{n_S} \left(S, \lambda_S^{JC}, J_S\mu_S^C\right)\right) J0^C \right\rangle = \qquad \text{(A.2)}
$$

$$
= \sum_{(',\,'')} \left| \left[1^3 \left(1^{n_0'} \left(I', \lambda_0', J_0'\mu_0'\right), 1^{n_S'} \left(S', \lambda_S', J_S'\mu_S'\right)\right) J'\mu'; \right. \right.
$$

$$
\left. \left. 1^3 \left(1^{n_0''} \left(I'', \lambda_0'', J_0''\mu_0''\right), 1^{n_S''} \left(S'', \lambda_S'', J_S''\mu_S''\right)\right) J''\mu'' \right] ISJ0^C \right\rangle
$$

$$
\left\langle \left(1^3 \left(1^{n_0'}1^{n_S'}\right); 1^3 \left(1^{n_0''}1^{n_S''}\right)\right) \mid \left(1^3, 1^3\right) 1^6 \underline{\left(1^{n_0}1^{n_S}\right)} \right\rangle
$$

$$
\left\langle \left(1^{n_0'} \left(I'\lambda_0'\right); 1^{n_0''} \left(I''\lambda_0''\right)\right) \mid \left(1^{n_0'}, 1^{n_0''}\right) 1^{n_0} \underline{\left(I\lambda_0\right)} \right\rangle
$$

$$
\left\langle \left(1^{n_S'} \left(S'\lambda_S'\right); 1^{n_S''} \left(S''\lambda_S''\right)\right) \mid \left(1^{n_S'}, 1^{n_S''}\right) 1^{n_S} \underline{\left(S\lambda_S\right)} \right\rangle
$$

$$
\left\langle \left(\lambda_0' J_0'\mu_0'; \lambda_0'' J_0''\mu_0''\right) \mid \left(\lambda_0'\lambda_0''\right) \lambda_0 \underline{\left(J_0\mu_0\right)} \right\rangle
$$

$$
\left\langle \left(\lambda_S' J_S'\mu_S'; \lambda_S'' J_S''\mu_S''\right) \mid \left(\lambda_S'\lambda_S''\right) \lambda_S \underline{\left(J_S\mu_S\right)} \right\rangle
$$

$$
\left\langle \left(J_0'J_S'\right) J', \left(J_0''J_S''\right) J'' \mid \left(J_0'J_0''\right) J_0, \left(J_S'J_S''\right) J_S \underline{\left(J\right)} \right\rangle
$$

$$
\left\langle \left(\mu_0'\mu_S'\right) \mu', \left(\mu_0''\mu_S''\right) \mu'' \mid \left(\mu_0'\mu_0''\right) \mu_0, \left(\mu_S'\mu_S''\right) \mu_S \underline{\left(0^C\right)} \right\rangle.
$$

Here n_0 (n_S) is the number of (u, d) (and s) quarks, I, S and J are the isospin, strangeness and total spin, respectively, the representation of SU_6^{JC} $\left(SU_3^C\right)$.

The complete expression of the expansion of the H dihyperon wave function (the H^* wave function was given in ref. [30]) is:

$$
|H\rangle = 0.867 \, |e_1\rangle - 0.499 \, |e_2\rangle, \qquad \text{(A.3)}
$$

$$|e_1\rangle = \sqrt{\frac{2}{15}}\frac{1}{\sqrt{2}}\left\{Q_0\left(\frac{1}{2},21,^4 21\right)Q_{-2}\left(1^{2,3}\,1^{2;4}\,21\right)-\longleftrightarrow\right\}-$$

$$-\sqrt{\frac{2}{15}}\frac{1}{\sqrt{2}}\left\{Q_0\left(\frac{1}{2},21,^2 21\right)Q_{-2}\left(1^{2,3}\,1^{2;2}\,21\right)-\longleftrightarrow\right\}+$$

$$+\sqrt{\frac{2}{15}}\frac{1}{\sqrt{2}}\left\{Q_0^N\left(\frac{1}{2},21,^2 0\right)Q_{-2}^\Xi\left(1^{2,3}\,1^{2;2}\,0\right)-\longleftrightarrow\right\}-$$

$$-\sqrt{\frac{1}{10}}\frac{1}{\sqrt{2}}\left\{Q_{-1}\left(1,1^{2,1}\,2;^2 21\right)Q_{-1}\left(1,1^{2,3}\,1^{2;2}\,21\right)+\longleftrightarrow\right\}+$$

$$+\sqrt{\frac{2}{45}}\left\{Q_{-1}^\Sigma\left(1,1^{2,3}\,1^{2;2}\,0\right)\right\}^2-\sqrt{\frac{1}{45}}\left\{Q_{-1}^{\Sigma^*}\left(1,1^{2,3}\,1^{2;4}\,0\right)\right\}^2+$$

$$+\sqrt{\frac{2}{45}}\left\{Q_{-1}\left(1,1^{2,3}\,1^{2;4}\,21\right)\right\}^2-\sqrt{\frac{4}{45}}\left\{Q_{-1}\left(1,1^{2,3}\,1^{2;2}\,21\right)\right\}^2-$$

$$-\sqrt{\frac{3}{10}}\frac{1}{\sqrt{2}}\left\{Q_{-1}\left(0,2,^1 1^{2;2}\,21\right)Q_{-1}\left(0,2,^3 2;^2 21\right)+\longleftrightarrow\right\},$$

$$|e_2\rangle = \sqrt{\frac{2}{5}}\frac{1}{\sqrt{2}}\left\{Q_0\left(\frac{1}{2},21,^2 21\right)Q_{-2}\left(1^{2,1}\,2;^2 21\right)-\longleftrightarrow\right\}+$$

$$+\sqrt{\frac{3}{20}}\left\{Q_{-1}\left(1,1^{2,1}\,2;^2 21\right)\right\}^2-\sqrt{\frac{1}{30}}\left\{Q_{-1}^\Sigma\left(1,1^{2,3}\,1^{2;2}\,0\right)\right\}^2-$$

$$-\sqrt{\frac{1}{15}}\left\{Q_{-1}^{\Sigma^*}\left(1,1^{2,3}\,1^{2;4}\,0\right)\right\}^2-\sqrt{\frac{1}{60}}\left\{Q_{-1}\left(1,1^{2,3}\,1^{2;2}\,21\right)\right\}^2-$$

$$-\sqrt{\frac{1}{30}}\left\{Q_{-1}\left(1,1^{2,3}\,1^{2;4}\,21\right)\right\}^2+\sqrt{\frac{1}{10}}\left\{Q_{-1}^\Lambda\left(0,2,^1 1^{2;2}\,0\right)\right\}^2+$$

$$+\sqrt{\frac{1}{20}}\left\{Q_{-1}\left(0,2,^1 1^{2;2}\,21\right)\right\}^2+\sqrt{\frac{1}{20}}\left\{Q_{-1}\left(0,2,^3 2;^2 21\right)\right\}^2+$$

$$+\sqrt{\frac{1}{10}}\left\{Q_{-1}\left(0,2,^3 2;^4 21\right)\right\}^2,$$

where

$$Q_0\left(\frac{1}{2},21,^2 21\right)=Q^3\left(n_0=3;I_0=\frac{1}{2},\lambda_0=21,J_0=\frac{1}{2},\mu_0=21\right),$$

$$Q_{-1}\left(0,2,^3 2;^2 21\right)=Q^3\left(n_0=2;I_0=0,\lambda_0=2,J_0=1,\mu_0=2;J=\frac{1}{2},\mu=21\right),$$

$$Q_{-2}\left(1^{2,3}\,1^{2;4}\,21\right)=Q^3\left(n_0=1;\lambda_S=1^2,J_S=1,\mu_S=1^2;J=\frac{3}{2},\mu=21\right).$$

Expression (A.3) and the analogous one for H^* from [30] are used to calculate matrix elements of the three-particle operator contained in $\Delta E_{\text{inst}}^{(3)}$. The scalar isospin components with the strangeness -1 : $Q_{-1}\left(0,2,^1 1^{2;2}\,21\right)$ and $Q_{-1}\left(0,2,^3 2;^2 21\right)$ only give a nonzero contribution. These components are weighted with the probability 25% in the total wave function.

To compute averages of two-particle operators, one may apply expressions of kind (A.2) or to construct the dissociation $q^6 \to q^4 \times q^2$. The coefficients of this dissociation may be easily found by using the equations for the Casimir operators of the SU_2^J, SU_3^F and SU_3^C groups. As a result, we have for the basis $SU_3^F \times SU_2^J \times SU_3^C$

$$\left|H\left(0^F,0^J,0^C\right)\right\rangle = \sqrt{\frac{1}{10}}q^4\left(2,0,2\right)q^2\left(2,0,2\right)+\sqrt{\frac{3}{10}}\left[q^4\left(1^2,0,1^2\right)q^2\left(1^2,0,1^2\right)+ \right. \tag{A.4}$$

$$\left. +q^4\left(1^2,1,2\right)q^2\left(1^2,1,2\right)+q^4\left(2,1,1^2\right)q^2\left(2,1,1^2\right)\right],$$

$$\left|H^*\left(21^F, 1^J, 0^C\right)\right\rangle = \sqrt{\frac{7}{60}} q^4\left(2, 0, 2\right) q^2\left(2, 0, 2\right) + \sqrt{\frac{13}{60}} q^4\left(1^2, 0, 1^2\right) q^2\left(1^2, 0, 1^2\right) + \tag{A.5}$$

$$+ \sqrt{\frac{17}{60}} q^4\left(1^2, 1, 2\right) q^2\left(1^2, 1, 2\right) + \sqrt{\frac{23}{60}} q^4\left(2, 1, 1^2\right) q^2\left(2, 1, 1^2\right).$$

If the dissociations (A.3)-(A.5) are expressed as

$$Q^6 = \sum_i w_i \left(Q^3\right)'_i \times \left(Q^3\right)_i = \sum_j u_j \left(Q^4\right)_j \times \left(Q^2\right)_j, \tag{A.6}$$

then the matrix elements of the three-(R_3) and two-(R_2) particle operators are calculated with the help

$$\left\langle Q^6 \left| R_3 \right| Q^6 \right\rangle = 20 \sum w_i^2 \left\langle Q_i^3 \left| R_3 \right| Q_i^3 \right\rangle, \tag{A.7}$$

$$\left\langle Q^6 \left| R_2 \right| Q^6 \right\rangle = 15 \sum u_j^2 \left\langle Q_j^2 \left| R_2 \right| Q_j^2 \right\rangle, \tag{A.8}$$

where the combinatorial factors take into account the multiple character of interaction in a n-particle antisymmetric states.

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